LIMITS TO THE EFFCIENCY OF IMAGING SYSTEMS

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ABSTRACT

The application of microelectronic techniques to the production of far infrared and submillimeter detectors has resulted in the design and production of large area arrays. It is of great importance to understand the optimal approach to optically couple these detector arrays to a telescope. In this paper, the performance of an ideal position sensitive detector and an ideal matched "Airy" feed are compared. With the development of fiber optics, this comparison is relevant for optical applications as well, because single mode fibers can be produced with spatially coherent coupling structures. We show that the sensitivity of the Airy feedhorn and the position sensitive detectors are identical for a given effective area, λ^2. For larger areas, the position sensitive array has higher sensitivity because the large focal plane area required by the feed results in an undersampling of the available modes. This undersampling must be recovered by scanning the instrument to faithfully reproduce the image.

Keywords: Submillimeter, Detectors, Bolometers, Cryogenics

INTRODUCTION

In this paper, we analyze the formation of an image from two complementary approaches: the classic radio and optical perspectives [1, 2]. From the "radio" point of view, the detection of an image produced by an optical system consists of determining the occupation numbers of each of a complete set of spatial modes that describe the field. Each of these spatial modes has an étendue, $A\Omega = \lambda^2$. In principle, one can produce an optical system that will concentrate all the energy from any single mode onto a detector of étendue λ^2. Such a filter is optimal for the detection of a point source of known position, that is, when the spatial phase of the feedhorn can be adjusted to match that of the source. The next case to consider is the sensitivity for a source of unknown position. In this case, multiple pointings are required of the feedhorn in order to allow the position of the source to be determined. Thus, the integration time for a given antenna pointing is increased as an expected consequence of the finite size of the single mode feed structure used to limit the angular acceptance.

Alternatively, one can consider a detector in the form of a continuous perfectly absorbing sheet in the focal plane. The illuminating solid angle is limited by a cryogenic stop in the instrument, effectively allowing only the light passing through the telescope to reach the focal plane. This absorbing sheet allows determination of the position of every photon by a pixelated array of detectors. In this case, each point in the focal plane is observing at all times. Each area of étendue λ^2 in the focal plane has sensitivity equal to that of an Airy horn fully sampling an area of étendue λ^2. An Airy horn would produce a "top hat" illumination pattern on the primary. Such a feed would require an infinitely wide aperture and is not available in practice, however, provides a useful mathematical figure of merit. The sensitivity limits of these two systems can be shown to be formally identical. The Airy feed horn provides high spatial efficiency with low temporal efficiency and noise from the full λ^2 étendue, while the continuous planar detector provides low spatial efficiency with high temporal efficiency and lower noise because of the smaller étendue in the elemental areas.

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POINT SOURCE OF *A PRIORI* KNOWN POSITION

We consider the case where the position of a point source is known; no additional observing time is required to determine it. A source imaged through a uniform circular aperture of diameter D produces an amplitude distribution in the focal plane, $E_a(u) = 2J_1(u)/u$, where $u = \pi\ r\ D/\ f_{eff}\ \lambda = \pi \sin\theta\ D/\lambda$, r is the radius, and f_{eff} is the effective focal length of the system. The aperture acts as a low-pass spatial filter with a cut-off angular frequency of ~D/λ, thus, the angular Nyquist sampling rate for a square tiling is ~λ/2D. Since the position of the point source is known, the information content of the image is fully specified by a single mode.

Coherent "Airy" Feed:

For a known source position, all available time can be used to measure the occupation number of this mode. To make this measurement, we must choose a spatial filter. The optimal spatial filter will combine the amplitudes from the entire aperture distribution in phase onto a single detector with étendue of λ^2. The coupled power can be expressed as,

$$\kappa_{af} = |\langle E_a | E_f \rangle|^2 / [\langle E_a | E_a\rangle \langle E_f | E_f \rangle] \tag{1}$$

For the matched system under consideration, $E_f = E_a(u)$, thus, the coupling is unity. The background power arises from the λ^2 throughput of the single mode feed. This measurement is the best one can do for the detection of a known source; one detects all the power in the single mode while observing the background power (and noise) from the étendue of a single mode, λ^2. For a Gaussian feed matched to the Airy pattern, the achievable coupling is ~0.82, while for a scalar feed one finds ~0.84. The signal to noise ratio per unit time in the "Airy" feed will be defined to be unity, and will be used as the point of reference for other cases.

Position Sensitive Planar Detector:

Next we consider the application of an optimal spatial filter to the distribution of photon arrivals on an ideal planar detector. In this case, the photons illuminating the screen produce an intensity distribution, $\propto E_a^2$. We assume that a cryogenic pupil stop is present in the system to limit the étendue of the detector to thermal radiation arising from the pupil of the illuminating system itself. The detection of a point source in this case consists of applying a spatial filter to the measured intensity distribution. For this case, we assume the noise is spatially uncorrelated, since the mode occupation numbers are typically much less than unity for submillimeter applications using cooled optics and achievable optical efficiencies. Knowing the spatial distribution of the signal and the noise, we can construct a Wiener optimal filter, which produces an estimate of the intensity of the source that is optimal in a least squares sense. In the case of our spatially white noise, the optimal filter is a matched filter; it has the same shape as the intensity distribution from the source itself, $E_p^2 = E_a(u)^2$. Therefore, the signal to noise ratio relative to an airy feed resulting from application of this filter to the point source intensity distribution is,

$$(S/N)|_{planar} = \{\langle E_a^2 | E_a^2 | 1\rangle/\langle E_a | E_a\rangle\} \ / \ |\langle 1 | E_a^4 | 1\rangle/\langle E_a | E_a\rangle|^{1/2} \tag{2}$$

The noise of this measurement is determined by applying the detector's weighting filter to the uniform background to determine the background power detected by the filter, and then determining the statistical fluctuations arising from this power. One finds the S/N is ~0.68 relative to ideal airy feed or ~0.81 relative to an achievable scalar feed design.

SOURCE OF UNKNOWN POSITION

For a point source with unknown source position or spatially extended features, more than one spatial mode is required to fully specify the information content of the image. Since the étendue of the single mode feedhorn is λ^2, we consider a region of the planar detector with an étendue of λ^2 for comparison. In both cases, we will assume that a source is at an unknown position within a region of $(\lambda/D)^2$. The observing strategy for the planar detector is to stare for the full observing time. The feedhorn will scan to uniformly sample this area during the integration time. The degree of coherence between two points separated by a distance of $\delta\zeta_{ij}$ in the image plane is,

$$\Lambda \equiv \langle E_i | E_j \rangle / | \langle E_i | E_i \rangle \langle E_j | E_j \rangle |^{1/2} = 2\, J_1(\delta u_{ij}) / \delta u_{ij}, \tag{3}$$

where $\delta u_{ij} = \pi\ \delta\zeta_{ij}\ D/\ f_{eff}\ \lambda$. In general, partial coherence is present between the two points in the image plane even though the source was incoherent. The interbeam correlations can be treated either in the aperture plane or beam domains. In the case of a feedhorn, to achieve a high coupling between the feed and the aperture, a relatively large area is used to define the beam which effectively reflects all but the desired spatial mode from the image plane. As a result, the spatial interbeam correlation will be small; however, in scanning the array to fully sample the image, the correlations between adjacent temporal samples will be large. For the planar array, the roles of the spatial and temporal correlations in the window function are effectively interchanged for Nyquist sampling of the planar detector in the image plane.

Coherent "Airy" Feed:

The signal to noise ratio for the above conditions will now be calculated for the feedhorn case. The time spent observing the elemental region dx^2 in the image plane is, $\tau = (dx/f_{eff})^2\ T$, where T is the scan duration. The efficiency of detecting the mode is 1.0, so the noise in the measurement is determined by the noise arising from the background over the λ^2 étendue and the noise per observation is $(F\lambda^2 h\nu/\tau)^{1/2}$, where F is the background power per unit area per steradian. Summing the weights for each observation of the source, the resulting signal to noise ratio achieved per scan is, $S/N(\text{feed}) = \langle\Lambda^2\rangle/(F\lambda^2 h\nu/T)^{1/2}$.

Position Sensitive Planar Detector:

Similarly, for the case of the planar position sensitive detector, since each region dx^2 in the image plane views the sky all the time, the integration time for all elements of the planar detector is T. The background noise arising from this elemental area is $(F\lambda^2 h\nu/T)^{1/2}$, so the signal to noise ratio from this area is $S/N(\text{planar}) = \langle\Lambda^2\rangle/(F\lambda^2 h\nu/T)^{1/2}$. This comparison confirms the intuition that by recording the arrival position of each photon in an image, one preserves all the information available in a temporally stationary image.

Extraction of Point Sources:

The actual detection of a point source requires the application of a spatial filter to the measurements in each of the elemental areas. Since the detector noise is not spatially correlated, the Wiener optimal filter will be the matched filter, having the same spatial distribution as the PSF of the telescope itself. Since the signal distribution and noise are the same in the two cases considered, the resulting measurement will be the same in both cases. In comparing data from the temporally sampled data to discretely spatially sampled data, the most significant differences arise from the fact that a pixelized detector produces discrete samples, while the temporally sampled horn output produces a continuous distribution (although in practice, it will be sampled at some frequency). The optimal filter for the extraction of a point source in the discretely sampled case depends on the spatial phase of the point source with respect to the pixels. The proper analysis of sampled data requires proper interpolation functions to avoid this problem. In actual operation, multiple dithered images will be taken, and the data will be interpolated onto a finer grid than the Nyquist sampled pixelization resolving, much of this problem [3,4,5].

In the limit that the noise is spatially uncorrelated, the optimal filter is the matched filter; this filter provides the best estimate of the brightness of a source. The improvement of this estimate over that of a single measurement centered on the source comes at a cost; the effective beam size for this measurement has a size determined by the convolution of the spatial filter with the PSF of the system, which is broader than the intrinsic PSF of the system. In general, one can produce filters that trade sensitivity for angular resolution. In the case where the limiting noise is set by spatial confusion, the optimal filter is different. In this case, the noise has a spatial correlation that is described by the Fourier transform of the PSF of the system. The optimal filter in this case must give higher weight to the higher spatial frequencies, producing a filtered image with better spatial resolution. The sensitivity of a system and its confusion limit are related. In general, one must quote sensitivities using detection filters that are optimal or suitable for the degree of confusion in the field. In particular, one cannot quote sensitivity using a Wiener filter for background limited operation, and then compute the confusion based on the instrumental PSF. One must use the PSF convolved with the chosen detection filter to evaluate the confusion.

Focal Plane Utilization:

Finally, we consider practical detector systems, comparing an array of close packed Nyquist sampled square pixels to an array of Gaussian feedhorns spaced by ~2 f# λ in the focal plane. We find that the efficiencies of these two systems per unit étendue are equal for sources of unknown position. The fully sampled array has the advantage for a given focal plane area by the underfilling factor of the array of feedhorns, which is approximately ~3.4 in this case. Thus, for cases where the feeds occupy all available space, the planar array can do better by this factor in overall observing speed.

One observes by choosing to apodize in the focal plane with a single mode horn, the space require to receive a subset of modes in the focal plane are rejected by the system, resulting in lower overall efficiency. If such apodization is required, it can be accomplished by using a graded cryogenic metallic sheet in the pupil plane designed to provide the required edge taper on the primary. This can then be sampled by the focal plane array with full sampling density, preserving the speed advantage of the filled array while providing the required edge taper on the primary. Such a system provides all the external stray light benefits of a feedhorn system, but does not suffer from the loss of observing speed from the underfilling of feeds in the focal plane.

SUMMARY

For isolated point sources of known position, the feedhorn has a significant advantage over a continuous detector, since in this case, one can spend all available time observing this mode. If the position needs to be confirmed, unknown or limited by confusion, we enter the limit where all positions must be observed with equal weight. In this case, the sensitivity of the ideal feedhorn and the ideal sheet detector are equal per unit étendue. However, the spatial filling factor possible with a practical feedhorns in the focal plane is only about ~0.3, resulting in a significant loss of efficiency. Thus, for applications limited by available focal plane space, the close packed arrays offer significant overall efficiency advantages of about a factor of ~3.4 over arrays of efficient feedhorns. The primary difference in the two cases is that the multiplexing is temporal in the case of the coherent feed and spatial in the case of the planar detector. As noted by Stein [6], the correlations of adjacent measurements on the field are the same, whether they are measured simultaneously in a position sensitive array, or by sequential sampling of the same elemental areas with a single mode detector. The net efficiency per elemental area of the image plane is the same in either case.

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